

# Adaptive Nonlinear Hierarchical Control of a Novel Quad Tilt-Wing UAV

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**Abstract**—Position control of a novel unmanned aerial vehicle SUAVI (Sabanci University Unmanned Aerial Vehicle) via a nonlinear hierarchical adaptive control approach is presented in this paper. The hierarchy consists of two levels. In the upper level, a model reference adaptive controller creates virtual control commands so as to make the UAV follow a given desired trajectory. The virtual control inputs are then converted to desired attitude angle references which are fed to the lower level attitude controller. Lower level controller is a nonlinear adaptive controller. The overall controller is developed for the full nonlinear dynamics of the tilt-wing UAV and thus no linearization is required. In addition, since the approach is adaptive, uncertainties in the UAV dynamics can be handled. Performance of the control approach is presented via simulation results.

## I. INTRODUCTION

The focus of this paper is a hybrid-wing type Unmanned Aerial Vehicle (UAV), which combines the advantages of the two other types, namely fixed-wing and rotary-wing UAVs: A hybrid UAV, like a rotary-wing UAV, can fly vertically without the need of any infrastructure for takeoff or landing and does not need any forward velocity for maneuvering. In addition, it can fly with high speed for extended periods of time, like a fixed-wing UAV. Tilt-rotor UAVs, a subgroup of hybrid designs, are attractive subjects of research due to their energy efficiency, stability and controllability [1], [2]. Under tilt-wing category, research studies can be found for dual tilt-rotor UAVs [3], [4] and quad tilt-wing UAVs [5], [6]. In general, the former type has the disadvantage of requiring cyclic control on propellers which adds to the mechanical complexity.

Tilt-wing UAVs are difficult to control due to many reasons: they have Multi Input Multi Output (MIMO) nonlinear dynamics. In addition, due to sources such as unpredictable damages and actuator malfunctions, uncertainty may pose a big challenge [7]. Furthermore, the variations in mass moments of inertia during tilting of the wings require compensation from the controller.

Among many proposed controller schemes in the literature, some recent ones are PD2 controller [8], LQ and PID controllers [9], LQR controller [10], thrust vectoring with a PID structure [11], sliding mode observers with feedback

linearization [12] and nonlinear control approaches [13], to name a few.

The above mentioned approaches showed promising successful results. However, they do not offer explicit uncertainty compensation or they do not provide enough robustness for large uncertainties such as structural damage. Literature contains controller proposals that actually overcome these issues via the employment of adaptive control. An approach that uses artificial neural networks (ANN) for the estimation of the nonlinear components of a quadrotor dynamics is presented in [14]. A study that discusses the advantage of the adaptive control over feedback linearization is provided in [15]. However, in [15], small attitude angles and slowly varying slack variables assumptions are used in the adaptive control design. An adaptive backstepping approach is presented in [16] but the uncertainty is assumed to be only in the mass of the quadrotor. MIT's quadrotor controller [7] has explicit uncertainty compensation via adaptation but a linear model is used in the design. Linear model assumption may introduce a limitation on the range of workable operating points and the performance outside of certain operating points may be unpredictable or degraded for the proposed approach.

In this work, nonlinear, hierarchical adaptive control of a novel quad tilt-wing UAV SUAVI (Sabanci university Unmanned Aerial Vehicle) is presented (see Fig. 1). SUAVI was previously designed, manufactured and flight tested by the co-author Unel and his students and earlier research results have been published about the aerodynamic and mechanical design, prototyping, control system design and flight tests [17]–[19]. In this work, different from authors' earlier research, a controller that explicitly compensates for the uncertainties in SUAVI's system dynamics is implemented. Similar to authors' earlier works, this controller has a hierarchical structure. However, in this paper a Model Reference Adaptive Controller (MRAC) [20] resides in the upper level and is responsible for creating virtual control inputs that would make the UAV follow a given trajectory. These virtual control inputs are realized by achieving certain UAV orientations, which is ensured by the lower level adaptive nonlinear controller [21]. This approach is based on full nonlinear dynamics of the tilt-wing UAV and therefore do not need linearization. In addition, the controller does not contain difficult to tune and computationally expensive components. Finally, adaptation provides explicit uncertainty compensation. This controller was recently proposed by the authors in [22] where an implementation on quadrotor dynamics (without wings) was presented. In this paper, the

\*This work was not supported by any organization

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controller is put through a much more challenging task of controlling SUAVI, which enjoys the properties of a quadrotor and a fixed wing UAV simultaneously, as explained above.

The organization of the paper is as follows. In Section II, the full nonlinear model of SUAVI is provided. The design of the hierarchical controller is given in Section III. Simulation results for two different scenarios are presented in Section IV and a summary is given in Section V.



Fig. 1: Quad tilt-wing UAV SUAVI with two different wing angle configurations

## II. SYSTEM MODEL

Nonlinear dynamics of the quad tilt-wing UAV is briefly described in this section. For details, see [17].

Using rigid body assumption, a generic unmanned air vehicle system dynamics is given as below:

$$\begin{bmatrix} mI_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_b \end{bmatrix} \begin{bmatrix} \dot{V}_w \\ \dot{\Omega}_b \end{bmatrix} + \begin{bmatrix} 0 \\ \Omega_b \times (I_b \Omega_b) \end{bmatrix} = \begin{bmatrix} F_t \\ M_t \end{bmatrix} \quad (1)$$

where  $m$  and  $I_b$  represent the mass and the inertia matrix in the body frame and  $V_w$  and  $\Omega_b$  represent the linear velocity with respect to world frame and the angular velocity with respect to body frame of the vehicle, respectively. The net force and the moment applied on the vehicle are represented by  $F_t$  and  $M_t$ , respectively (see Fig. 2). It is noted that for tilt-wing quadrotors, these forces and moments are functions of the rotor thrusts and wing angles.

Using vector-matrix notation, (1) can be rewritten as follows:

$$M\dot{\zeta} + C(\zeta)\zeta = G + O(\zeta)\omega + E(\xi)\omega^2 + W(\xi) \quad (2)$$

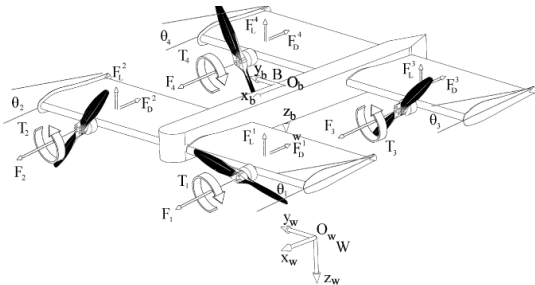


Fig. 2: Forces and moments on the UAV.

where,

$$\zeta = [\dot{X}, \dot{Y}, \dot{Z}, p, q, r]^T \quad (3)$$

and

$$\xi = [X, Y, Z, \Phi, \Theta, \Psi]^T, \quad (4)$$

where  $X, Y$  and  $Z$  are the coordinates of the center of mass with respect to the world frame,  $p, q$  and  $r$  are the angular velocities in the body frame and  $\Phi, \Theta$  and  $\Psi$  are the roll, pitch and yaw angles of the vehicle expressed in the world frame.  $M$ , the inertia matrix,  $C$ , Coriolis-centripetal matrix and  $G$ , the gravity term, are given as follows:

$$M = \begin{bmatrix} mI_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & \text{diag}(I_{xx}, I_{yy}, I_{zz}) \end{bmatrix} \quad (5)$$

$$C(\zeta) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{zz}r & -I_{yy}q \\ 0 & 0 & 0 & -I_{zz}r & 0 & I_{xx}p \\ 0 & 0 & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix} \quad (6)$$

$$G = [0, 0, mg, 0, 0, 0]^T \quad (7)$$

where,  $I_{xx}, I_{yy}$  and  $I_{zz}$  are the moments of inertia around the body axes. The gyroscopic term,  $O(\zeta)\omega$ , is given as

$$O(\zeta)\omega = J_{prop} \left( \sum_{i=1}^4 [\eta_i \Omega_b \times \begin{bmatrix} c_{\theta_i} \\ 0 \\ -s_{\theta_i} \end{bmatrix} \omega_i \right) \quad (8)$$

where,  $\eta_{(1,2,3,4)} = 1, -1, -1, 1$  and  $c_{\theta_i}$  and  $s_{\theta_i}$  represent cosine and sine of the wing angles, respectively. When two simplifying assumptions are used, namely neglecting the aerodynamic downwash effect of the rear wings and using same angles for the front and rear wings, system actuator vector,  $E(\xi)\omega^2$ , can be given as

$$E(\xi)\omega^2 = \begin{bmatrix} (c_{\Psi}c_{\Theta}c_{\theta_f} - (c_{\Phi}s_{\Theta}c_{\Psi} + s_{\Phi}s_{\Psi})s_{\theta_f})u_1 \\ (s_{\Psi}c_{\Theta}c_{\theta_f} - (c_{\Phi}s_{\Theta}s_{\Psi} - s_{\Phi}c_{\Psi})s_{\theta_f})u_1 \\ (-s_{\Theta}c_{\theta_f} - c_{\Phi}c_{\Theta}s_{\theta_f})u_1 \\ s_{\theta_f}u_2 - c_{\theta_f}u_4 \\ s_{\theta_f}u_3 \\ c_{\theta_f}u_2 + s_{\theta_f}u_4 \end{bmatrix} \quad (9)$$

where,  $\theta_f$  represents front wing angle. Inputs  $u_1, u_2, u_3$  and  $u_4$  in (9) are given as:

$$u_1 = k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (10)$$

$$u_2 = kl_s(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (11)$$

$$u_3 = kl_l(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \quad (12)$$

$$u_4 = k\lambda(\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_4^2) \quad (13)$$

where,  $k, l_s, l_l$  and  $\lambda$  are the motor thrust constant, rotor distance to center of mass along y axis, rotor distance to center of mass along x axis and torque/force ratio, respectively.

The wing forces  $W(\xi)$ , lift and drag, and the moments they create on the UAV are given as

$$W(\zeta) = \begin{bmatrix} R_{bw} \begin{bmatrix} F_D^1 + F_D^2 + F_D^3 + F_D^4 \\ 0 \\ F_L^1 + F_L^2 + F_L^3 + F_L^4 \\ 0 \end{bmatrix} \\ l_l(F_L^1 + F_L^2 - F_L^3 - F_L^4) \\ 0 \end{bmatrix} \quad (14)$$

where,  $F_D^i = F_D^i(\theta_f, v_x, v_z)$  and  $F_L^i = F_L^i(\theta_f, v_x, v_z)$ . The relationship between the linear velocities in body frame  $v_x, v_y, v_z$  and linear velocities in the world frame  $\dot{X}, \dot{Y}, \dot{Z}$  are given as

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = R_{wb}^T(\Phi, \Theta, \Psi) \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}. \quad (15)$$

Using (1), the following rotational dynamics, that is in a form suitable for attitude controller design, is obtained:

$$M(\alpha_w)\dot{\Omega}_w + C(\alpha_w, \Omega_w)\Omega_w = E^T M_t \quad (16)$$

where,  $\alpha_w = [\Phi, \Theta, \Psi]^T$ ,  $\Omega_w = [\dot{\Phi}, \dot{\Theta}, \dot{\Psi}]$  and  $E(\alpha_w)$  is the velocity transformation matrix, which is given as

$$E(\alpha_w) = \begin{bmatrix} 1 & 0 & -s_\Theta \\ 0 & c_\Phi & s_\Phi c_\Theta \\ 0 & -s_\Phi & c_\Phi c_\Theta \end{bmatrix}. \quad (17)$$

The relationship between the angular velocity of the UAV in the body frame,  $\Omega_b$ , and in the world frame,  $\Omega_w$ , is given as

$$\Phi_b = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = E(\alpha_w)\Omega_w. \quad (18)$$

The modified inertia matrix  $M(\alpha_w)$  in (16) is given as

$$M(\alpha_w) = \begin{bmatrix} I_{xx} & 0 & -I_{xx}s_\Theta \\ 0 & I_{yy}c_\Phi^2 + I_{zz}s_\Phi^2 & M_{23} \\ -I_{xx}s_\Theta & M_{23} & M_{33} \end{bmatrix} \quad (19)$$

where,

$$M_{23} = I_{yy}c_\Phi s_\Phi c_\Theta - I_{zz}c_\Phi s_\Phi c_\Theta \quad (20)$$

$$M_{33} = I_{xx}s_\Theta^2 + I_{yy}s_\Phi^2 c_\Theta^2 + I_{zz}c_\Phi^2 c_\Theta^2 \quad (21)$$

and the Coriolis Matrix,  $C(\alpha_w, \Omega_w)$  is given as

$$C(\alpha_w, \Omega_w) = \begin{bmatrix} 0 & C_{12} & C_{13} \\ I_{xx}d & I_{yy}f + I_{zz}g & C_{23} \\ I_{xx}e & I_{yy}h + I_{zz}k & C_{33} \end{bmatrix}. \quad (22)$$

In (22),  $C_{ij}$ s are defined as

$$C_{12} = -I_{yy}s_3c_\Phi - I_{zz}s_2s_\Phi \quad (23)$$

$$C_{13} = -I_{xx}c_\Theta\dot{\Theta} - I_{yy}s_3s_\Phi c_\Theta + I_{zz}s_2c_\Phi c_\Theta \quad (24)$$

$$C_{23} = I_{xx}mm + I_{yy}n + I_{zz}p \quad (25)$$

$$C_{33} = I_{xx}q + I_{yy}r + I_{zz}\epsilon, \quad (26)$$

where,

$$s_1 = \dot{\Phi} - s_\Theta\dot{\Psi} \quad (27)$$

$$s_2 = c_\Phi\dot{\Theta} + s_\Phi c_\Theta\dot{\Psi} \quad (28)$$

$$s_3 = -s_\Phi\dot{\Theta} + c_\Phi c_\Theta\dot{\Psi} \quad (29)$$

$$d = s_3c_\Phi + s_2s_\Phi \quad (30)$$

$$e = s_3s_\Phi c_\Theta - s_2c_\Phi c_\Theta \quad (31)$$

$$f = -s_\Phi\dot{\Phi}c_\Phi - s_1c_\Phi s_\Phi \quad (32)$$

$$g = s_1s_\Phi c_\Phi + c_\Phi\dot{\Phi}s_\Phi \quad (33)$$

$$h = s_3c_\Phi s_\Theta - s_\Phi^2\dot{\Phi}c_\Theta + s_1c_\Phi^2c_\Theta \quad (34)$$

$$k = s_2s_\Phi s_\Theta + s_1s_\Phi^2c_\Theta - c_\Phi^2\dot{\Phi}c_\Theta \quad (35)$$

$$mm = -s_3s_\Theta c_\Phi - s_2s_\Theta s_\Phi \quad (36)$$

$$a = c_\Phi\dot{\Phi}c_\Theta - s_\Phi s_\Theta\dot{\Theta} \quad (37)$$

$$n = ac_\Phi - s_1s_\Phi^2c_\Theta \quad (38)$$

$$b = -s_\Phi\dot{\Phi}c_\Theta - c_\Phi s_\Theta\dot{\Theta} \quad (39)$$

$$p = -s_1c_\Phi^2c_\Theta - bs_\Phi \quad (40)$$

$$q = c_\Theta\dot{\Theta}s_\Theta - s_3s_\Theta s_\Phi c_\Theta + s_2s_\Theta c_\Phi c_\Theta \quad (41)$$

$$r = s_3s_\Phi c_\Theta s_\Theta + as_\Phi c_\Theta + s_1s_\Phi c_\Theta^2 c_\Phi \quad (42)$$

$$\epsilon = -s_2c_\Phi c_\Theta s_\Theta - s_1c_\Phi c_\Theta^2 s_\Phi + bc_\Phi c_\Theta \quad (43)$$

### III. CONTROLLER DESIGN

To control the position of SUAVI, whose nonlinear dynamics is provided in Section II, a hierarchical nonlinear control approach is used that can adapt its parameters online. On the upper level, a Model Reference Adaptive Controller (MRAC) [20] provides virtual control inputs to control the position of the UAV. These control inputs are converted to desired attitude angles which are then fed to the lower level attitude controller. A nonlinear adaptive controller [21] is employed as the attitude controller so that uncertainties can be compensated without the need for linearization of system dynamics. Closed loop control system structure is presented in Fig. 3 and upper and lower level controllers are described below.

#### A. MRAC Design

A Model Reference Adaptive Controller (MRAC), that resides in the upper level of the hierarchy, is designed to control the position of the SUAVI, assuming that the system is a simple mass. This controller calculates the required forces that need to be created, by the lower level nonlinear controller, in the  $X, Y$  and  $Z$  directions, to make the UAV follow the desired trajectory. No information is used about the actual mass of the UAV during the design and this uncertainty in the mass is handled by online modification of control parameters based on the trajectory error. It is noted that the uncertainties in moment of inertia are handled by the lower level attitude controller, which is explained in the next section.

Consider the following system dynamics:

$$\dot{\underline{X}} = A\underline{X} + B_n\Lambda(u + D) \quad (44)$$

$$y = C\underline{X}, \quad (45)$$

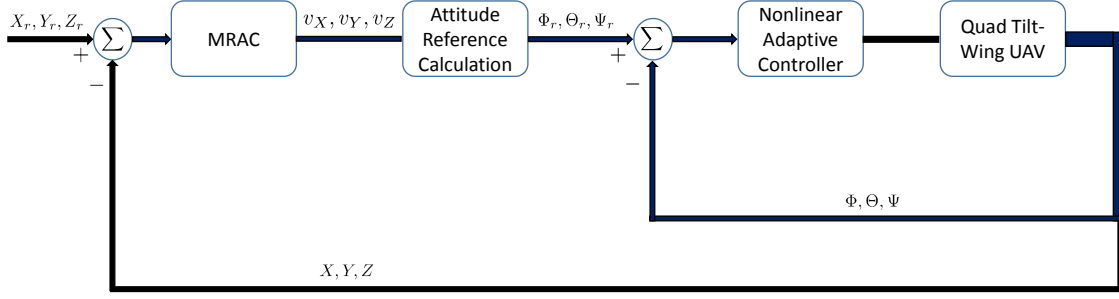


Fig. 3: Closed loop control system block diagram.

where,  $\underline{X} = [X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}]^T$ ,  $y$  is the plant output,

$$A = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \quad (46)$$

$$B_n = \begin{bmatrix} 0_{3 \times 3} \\ I_{3 \times 3} \end{bmatrix} \frac{1}{m_n} \quad (47)$$

$$\Lambda = \frac{m_n}{m} \quad (48)$$

$$D = \begin{bmatrix} 0_{5 \times 1} \\ mg \end{bmatrix}, \quad (49)$$

where  $m$  is the actual mass of the UAV that is assumed to be unknown,  $m_n$  is the nominal mass,  $g$  is the gravitational acceleration and  $\Lambda$  represents the uncertainty in the system.

1) *Reference Model Design:* Consider the following control law, which is to be used for the nominal system dynamics ( $\Lambda = 1$ ).

$$u_n = K_x^T \underline{X} + K_r^T r - D \quad (50)$$

where  $r \in R$ ,  $K_x \in R^{6 \times 3}$  and  $K_r \in R^{3 \times 3}$  are the reference input, control gain for the states and control gain for the reference input, respectively. Substituting (50) into (44), the nominal closed loop dynamics is obtained, which is given below:

$$\dot{\underline{X}}_n = (A + B_n K_x^T) \underline{X}_n + B_n K_r^T r. \quad (51)$$

In (51),  $K_x$  can be determined by any linear control design method, such as pole placement or LQR. Defining  $A_m = A + B_n K_x^T$ , nominal plant output is obtained as

$$y_n = C(sI - A_m)^{-1} B_n K_r^T r. \quad (52)$$

For a constant  $r$ , the steady state plant output can be calculated as

$$y_{ss} = -C A_m^{-1} B_n K_r^T r. \quad (53)$$

Using  $K_r^T = -(C A_m^{-1} B_n)^{-1}$ , it is obtained that

$$\lim_{t \rightarrow \infty} (y_n - r) = 0. \quad (54)$$

As a result, the reference model dynamics is determined as

$$\dot{\underline{X}}_m = A_m \underline{X}_m + B_m r \quad (55)$$

where,

$$A_m = A + B_n K_x^T \quad (56)$$

and

$$B_m = B_n K_r^T \quad (57)$$

$$= -B_n (C A_m^{-1} B_n)^{-1}. \quad (58)$$

2) *Adaptive Controller Design:* Consider the following adaptive controller:

$$u_{MRAC} = \hat{K}_x^T \underline{X} + \hat{K}_r^T r + \hat{D} \quad (59)$$

with the adaptive laws

$$\dot{\hat{K}}_x = -\Gamma_x \underline{X} e^T P B_n, \quad (60)$$

$$\dot{\hat{K}}_r = -\Gamma_r r e^T P B_n, \quad (61)$$

$$\dot{\hat{D}} = -\Gamma_d e^T P B_n \quad (62)$$

where  $e = \underline{X} - \underline{X}_m$ ,  $\Gamma_x, \Gamma_r, \Gamma_d$  are adaptive gains and  $P$  is the symmetric solution of the Lyapunov equation

$$A_m^T P + P A_m = -Q \quad (63)$$

where  $Q$  is a positive definite matrix. It can be shown [23] that the controller described in (59) - (63) provides convergence of the plant (44) and reference model (44) states in a stable manner, while keeping all the signals bounded.

### B. Attitude Reference Calculation

From (1) and (9), we obtain that

$$m\ddot{X} = (c_\psi c_\theta c_{\theta_f} - (c_\Phi s_\theta c_\psi + s_\Phi s_\psi) s_{\theta_f}) u_1 \quad (64)$$

$$m\ddot{Y} = (s_\psi c_\theta c_{\theta_f} - (c_\Phi s_\theta s_\psi - s_\Phi c_\psi) s_{\theta_f}) u_1 \quad (65)$$

$$m\ddot{Z} = (-s_\theta c_{\theta_f} - c_\Phi c_\theta s_{\theta_f}) u_1 + mg. \quad (66)$$

Right hand sides of (64)-(66) correspond to the forces determined by the MRAC position controller designed in the previous subsection:

$$u_{MRAC}^1 = (c_\psi c_\theta c_{\theta_f} - (c_\Phi s_\theta c_\psi + s_\Phi s_\psi) s_{\theta_f}) u_1 \quad (67)$$

$$u_{MRAC}^2 = (s_\psi c_\theta c_{\theta_f} - (c_\Phi s_\theta s_\psi - s_\Phi c_\psi) s_{\theta_f}) u_1 \quad (68)$$

$$u_{MRAC}^3 = (-s_\theta c_{\theta_f} - c_\Phi c_\theta s_{\theta_f}) u_1. \quad (69)$$

It is important to note that the  $D$  term in (44) addresses the gravitational force  $m.g$ . From (67)-(69), it is obtained that

$$u_1 = \sqrt{(u_{MRAC}^1)^2 + (u_{MRAC}^2)^2 + (u_{MRAC}^3)^2} \quad (70)$$

$$\Phi_d = \arcsin\left(\frac{-\rho_1}{u_1 s_{\theta_f}}\right) \quad (71)$$

$$\Theta_d = \left(\frac{-u_{MRAC}^3 u_1 c_{\theta_f} - u_1 \rho_2 s_{\theta_f} c_{\Phi_d}}{(\rho_2)^2 + (u_{MRAC}^3)^2}\right) \quad (72)$$

where,

$$\rho_1 = u_{MRAC}^1 s_{\Psi_d} - u_{MRAC}^2 c_{\Psi_d} \quad (73)$$

$$\rho_2 = u_{MRAC}^1 c_{\Psi_d} + u_{MRAC}^2 s_{\Psi_d}. \quad (74)$$

It is noted that, different from similar works in the literature, the desired attitude angles are functions of the wing angles.  $\Psi_d$ , the desired yaw angle, can be chosen by the UAV operator that would be appropriate for the mission at hand. These required attitude angles are given to the lower level attitude controller as references. The nonlinear adaptive attitude controller is described in the next section.

### C. Nonlinear Adaptive Control Design

To force the UAV follow the requested attitude angles, in the presence of uncertainties, a nonlinear adaptive controller [24] is employed. Defining  $u' = E^T M_t$ , (16) can be rewritten as

$$M(\alpha_w) \dot{\Omega}_w + C(\alpha_w, \Omega_w) \Omega_w = u'. \quad (75)$$

Equation (75), which describes the rotational dynamics of SUAVI, can be parameterized in a way such that the moment of inertia of the UAV,  $I_{UAV} = [I_{xx}, I_{yy}, I_{zz}]^T$ , appears linearly. This transformation is needed so that the uncertain moment of inertia terms appears in a form that is suitable for the adaptive control design:

$$Y(\alpha_w, \dot{\alpha}_w, \ddot{\alpha}_w) I_{UAV} = u'. \quad (76)$$

Consider the following definition

$$s = \dot{\tilde{\alpha}}_w + \Lambda_s \tilde{\alpha}_w \quad (77)$$

where  $\tilde{\alpha}_w = \alpha_w - \alpha_{wd}$ ,  $\alpha_{wd}$  is the desired value of  $\alpha_w$  and  $\Lambda_s \in R^{3 \times 3}$  is a symmetric positive definite matrix. Equation (77) can be modified as

$$s = \dot{\alpha}_w - \dot{\alpha}_{wr} \quad (78)$$

where

$$\dot{\alpha}_{wr} = \dot{\alpha}_{wd} - \Lambda_s \tilde{\alpha}_w. \quad (79)$$

A matrix  $Y' = Y'(\alpha_w, \dot{\alpha}_w, \dot{\alpha}_{wr}, \ddot{\alpha}_{wr})$  can be defined, to be used in linear parameterization, as in the case of (76), such that

$$M(\alpha_w) \ddot{\alpha}_{wr} + C(\alpha_w, \Omega_w) \dot{\alpha}_{wr} = Y'(\alpha_w, \dot{\alpha}_w, \dot{\alpha}_{wr}, \ddot{\alpha}_{wr}) I_{UAV}. \quad (80)$$

It can be shown that the following nonlinear controller,

$$u_{Nadp} = Y' \hat{I}_{UAV} - K_D s \quad (81)$$

where  $K_D \in R^{3 \times 3}$  is positive definite matrix and  $\hat{I}$  is an estimate of the uncertain parameter  $I$ , with an adaptive law

$$\dot{\hat{I}}_{UAV} = -\Gamma_I Y'^T s \quad (82)$$

where  $\Gamma_I$  is the adaptation rate, stabilizes the closed loop system and makes the error  $\tilde{\alpha}_w$  converge to zero.

The total thrust  $u_1$  is provided in (70). The rest of the control inputs in (9) can be calculated [17] by first defining  $u'' = (E(\alpha_w)^T)^{-1} u'$  and performing the following operations:

$$u_3 = \frac{u''_2}{s_{\theta_f}} \quad (83)$$

$$\begin{bmatrix} u_2 \\ u_4 \end{bmatrix} = \begin{bmatrix} s_{\theta_f} & -c_{\theta_f} \\ c_{\theta_f} & s_{\theta_f} \end{bmatrix}^{-1} \begin{bmatrix} u''_1 \\ u''_3 \end{bmatrix}. \quad (84)$$

Once these control inputs are determined, the thrusts created by the rotors can be calculated using linear relationships given in (10)-(13).

## IV. SIMULATION RESULTS

Simulation results for two different scenarios are presented in this section, where the nonlinear adaptive hierarchical controller is implemented using the full nonlinear dynamics of SUAVI which is provided in Section II. The reference model for the design of Model Reference Adaptive Controller is determined using an LQR with  $Q = \text{diag}([100, 100, 100])$  and  $R = \text{diag}([1, 1, 10])$ . It is assumed that UAV mass is uncertain with a 20% uncertainty. For the mass moment of inertias, 100% uncertainty is assumed, meaning that no prior information for these variables,  $I_{xx}, I_{yy}, I_{zz}$ , are used in the controller design. This is modeled by assigning zero to all initial conditions of the adaptive parameters. Nominal system dynamics parameters for SUAVI are provided in Table I.

TABLE I: UAV parameters

m	Mass of the UAV	4.5 kg
$l_s$	Distance of the rotor to the center of mass along y axis	0.3 m
$l_l$	Distance of the rotor to the center of mass along x axis	0.3 m
$I_{xx}$	Moment of inertia about x axis	0.405 kgm <sup>2</sup>
$I_{yy}$	Moment of inertia about y axis	0.405 kgm <sup>2</sup>
$I_{zz}$	Moment of inertia about z axis	0.72 kgm <sup>2</sup>
$\lambda$	Motor torque/force ratio	0.01 Nm/N

### A. First scenario

In the first scenario, SUAVI takes off vertically with 90-degree wing angles, which means that the wings are also in vertical position (wing angles are measured from the roll axis of the body frame). After finalizing the take-off at 10 meters above the ground, at t=10 seconds, SUAVI tilts its wings from 90 degrees to 20 degrees, while hovering at constant altitude. This tilting action takes 10 seconds. After wings reach their final position of 20 degrees, the UAV flies in the X direction for 150 meters, stops, and tilts its wings from 20 degrees back to 90 degrees, while still hovering, and gets

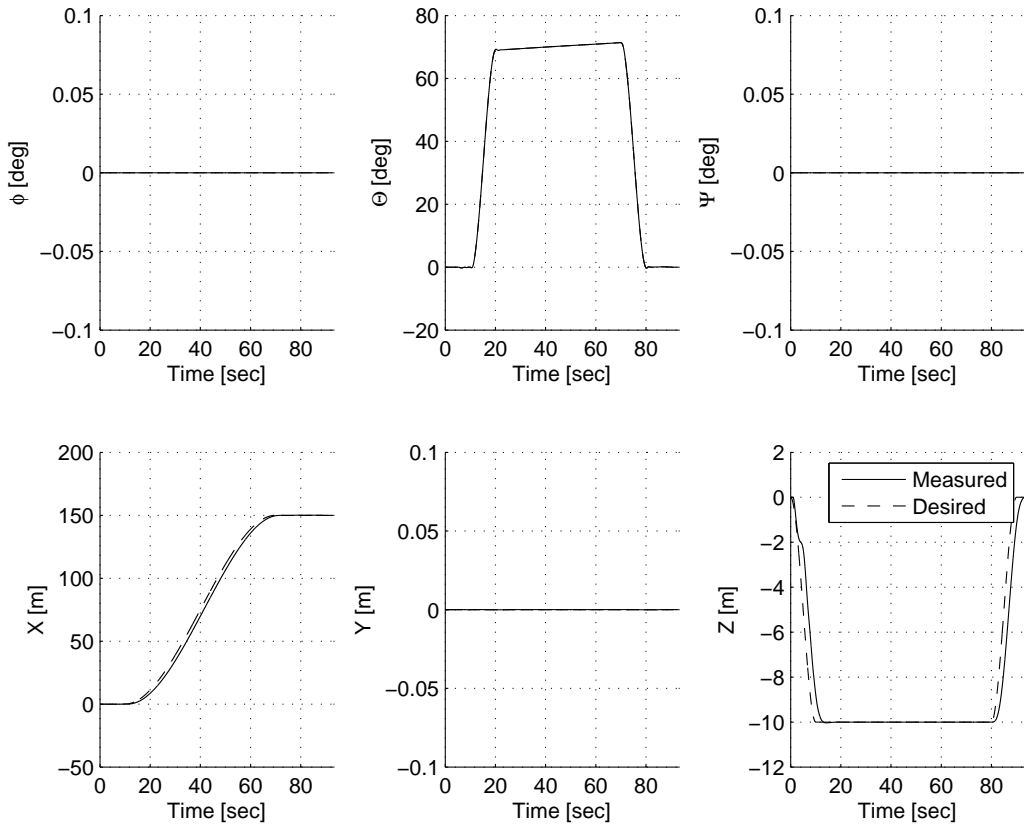


Fig. 4: Tracking curves for the attitude and the position of the UAV for Scenario 1.

ready for the landing. Finally, SUAVI performs a landing in 10 seconds while the wings are in vertical position.

Figure 4 shows the tracking performances for the attitude angles, roll, pitch and yaw, together with X, Y and Z positions, which shows that the employed nonlinear controller behaves as expected. It is noted that the change in pitch between  $t=10$  seconds and  $t=20$  seconds are due to the tilting of the wings, which is presented in Fig 5. As the wings are tilted, the inner loop controller changes the pitch to keep the UAV hovering without moving in X or Y directions.

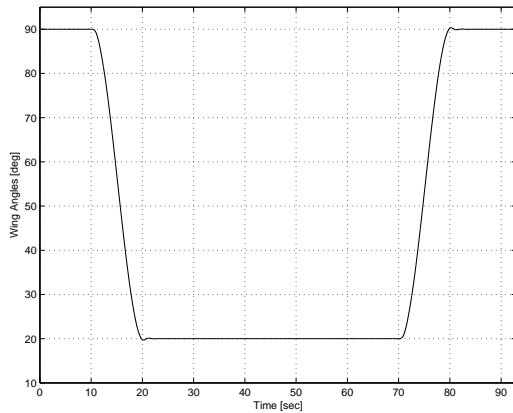


Fig. 5: Evolution of wing angles with time.

are smooth and they vary within a reasonable range.

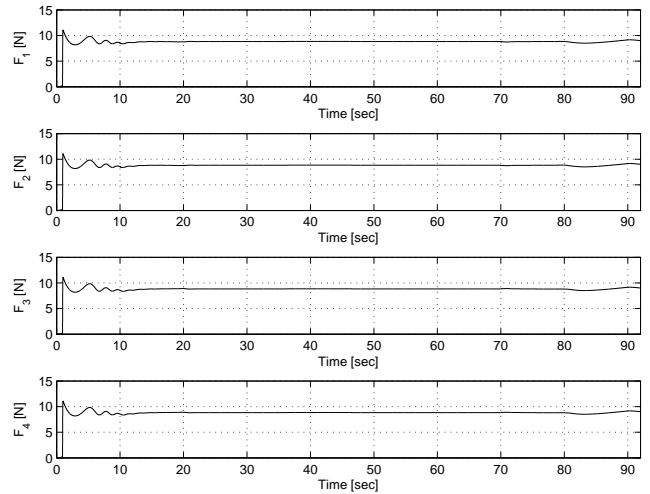


Fig. 6: Motor thrusts at Scenario 1.

### B. Second Scenario

In the second scenario, the UAV makes a vertical take-off, again with 90-degree wing angles, stops at  $Z=-10$  meters and tilts its wings from 90 degrees to 20 degrees while hovering at the same spot. With 20-degree wing angles, the UAV flies approximately 30 meters in the X direction and then follows a circular trajectory, while still at  $Z=-10$  meters, with a radius

Rotor thrusts are presented in Fig. 6, showing that they

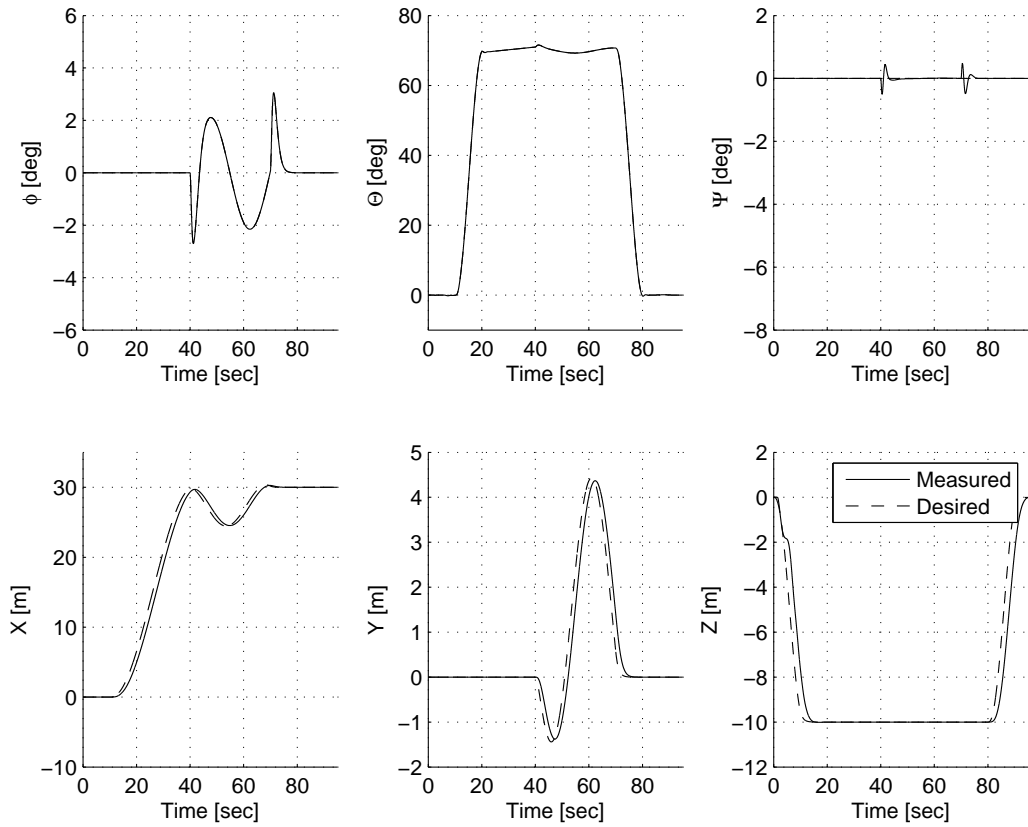


Fig. 7: Tracking curves for the attitude and the position of the UAV for Scenario 2.

of 3 meters. After completing the circle, SUAVI tilts its wings from 20 degrees back to 90 degrees and performs a vertical landing while also keeping its wings vertical.

Figure 7 shows the tracking performance of the controller for the position and attitude loops, which confirms that the orientation and the position of the UAV follows their desired values with reasonable speed. As in Scenario 1, the pitch angle change starting at  $t=10$  seconds are due to the tilting of the wings (same as in Fig 5) and the inner loop controller's effort to keep the UAV hovering at the same spot without moving laterally during this wing movement.

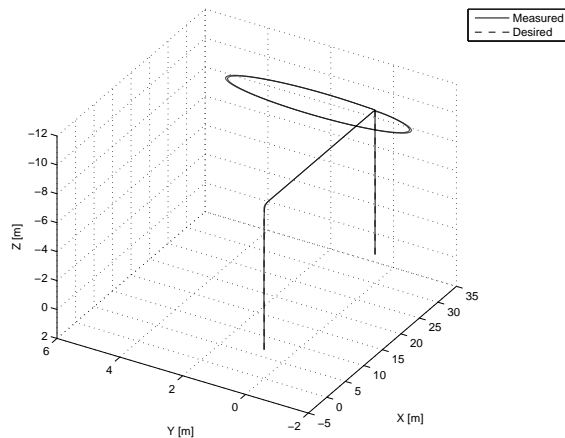


Fig. 8: 3-D trajectory in scenario 2.

Figure 8 shows the complete trajectory in a 3-dimensional (3D) graph. A close-up for the circular part of the trajectory is presented in Figure 9.

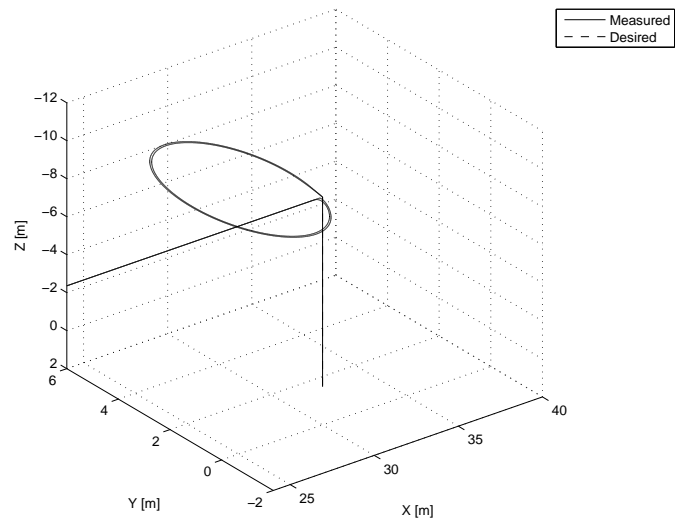


Fig. 9: 3-D trajectory zoomed at the circular motion in scenario 2.

Generated thrusts by the rotors are shown in Fig. 10, where it is seen that thrust variations are smooth and in a reasonable range.

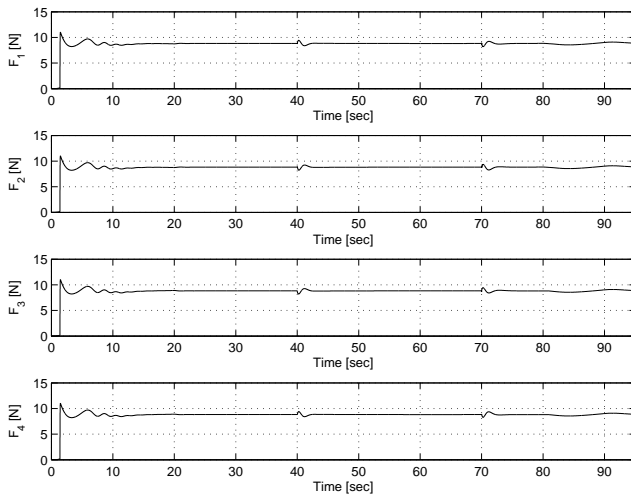


Fig. 10: Motor thrusts at Scenario 2.

## V. SUMMARY

In this paper, the implementation of a recently proposed nonlinear hierarchical adaptive controller on a new quad tilt-wing UAV with uncertain dynamics has been presented. The controller consists of two levels, where a model reference adaptive controller is at the higher level determining necessary forces to make the UAV follow a given trajectory, and a nonlinear adaptive controller is at the lower level making sure that the orientation of the UAV is adjusted properly to produce these forces requested by the upper level controller. Full nonlinear dynamics is utilized in the controller design without any linearizations, so the resulting control structure is not limited to operate around certain operating points. In addition, thanks to a hierarchical design, the position and attitude controllers can be constructed independently providing flexibility to the control engineer. Finally, with the help of online control parameter adjustment, uncertainties in the mass and moments of inertia can be handled without the need for parameter estimation. Two different flight scenarios were simulated and results of the simulations are quite promising. Variations in system dynamics due to the tilting of the wings and the ability of the adaptive controller to compensate these variations will be investigated in future research studies.

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